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GRAPH-CUT METHODS FOR GRAIN BOUNDARY SEGMENTATION (PREPRINT)

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Graph-Cut Methods for Grain Boundary Segmentation

Song Wang, Jarrell Waggoner, Jeff Simmons

Abstract: *This paper reviews the recent progress on using graph-cut methods for image segmentation, and discusses their applications to segmenting grain boundaries from materials science images.*

Introduction

The size and shape of crystals (i.e., grains) in polycrystalline materials (e.g., metals and metal alloys) are among the strongest determinants of many material properties, such as mechanical strength or fracture resistance. In materials science research, the emerging practice involves constructing 3D models of the grain structure and then applying finite element modeling to infer the mechanical properties from that structure [8, 10]. Accurately and automatically segmenting the grain boundaries from various material images can substantially facilitate the modeling and analysis of the material microstructure, and shorten the period of design and development of new materials.

Image segmentation is a fundamental problem in computer vision and image processing. In past decades, many image segmentation algorithms and tools have been developed and are used to process images in different domains, such as pictures taken indoors and outdoors, aerial images, medical images, and videos. Particularly, graph-cut methods for image segmentation have attracted tremendous interest in the computer vision community in recent years. Compared to classical image-segmentation methods, such as edge detection, region splitting/merging, and pixel clustering, graph-cut methods are more "global" by considering well-defined, comprehensive segmentation cost functions and seeking their globally optimal solutions using advanced graph theory.

Graph and Minimum Cut

In graph-cut methods, a graph $G = (V, E)$ with vertices V and edges E is first constructed to represent an image. Considering a 2D image, we can construct a vertex for each pixel and an edge between two vertices corresponding to two neighboring pixels, as shown in Fig. 1. A *graph cut* divides the graph into two subgraphs G_1 and G_2 by removing all the edges connecting G_1 and G_2. Two examples are shown in Fig. 1(b) and (d), where the removal of the edges intersected by the red dashed curve constitutes a graph cut. A graph cut corresponds to a segmentation boundary (either open or closed) in the image. Multiple-region image segmentation can be obtained by repeated graph cuts on the subgraphs G_1 and G_2.

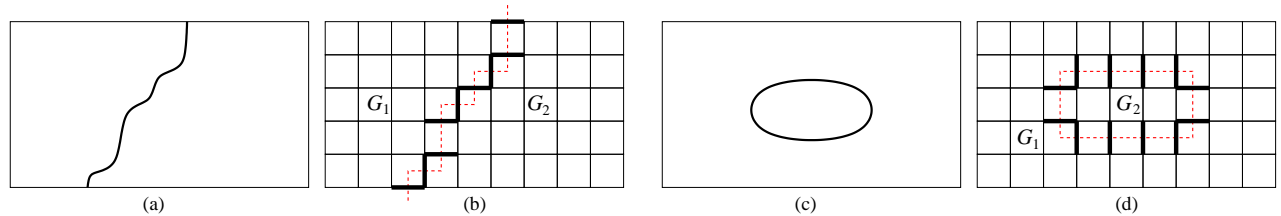

(a)   (b)   (c)   (d)

Figure 1: Image segmentation and graph cuts. (a) An image segmented by an open boundary. (b) The graph model for (a) and the graph cut for its segmentation boundary. (c) An image segmented by a closed boundary. (d) The graph model for (c) and the graph cut for its segmentation boundary.

Image-intensity information is typically encoded into an edge-weight function $w(u, v)$, where $(u, v) \in E$. For example, we can define $w(u, v)$ as a decreasing function of the intensity difference between vertices (pixels) u and v. This way, a graph cut that removes low-weight edges is more preferred for image segmentation. In graph-cut methods, the two central problems are: 1) defining a cost function for each possible graph cut to reflect the aforementioned preference, and 2) developing a graph algorithm to find the optimal graph cut that minimizes this cost function. In [15], the cost function is defined to be the total weight of the removed edges,

i.e.,

$$c(G_1, G_2) = \sum_{u \in G_1, v \in G_2, (u,v) \in E} w(u,v).$$

This is the well-known minimum cut problem, and its global optima can be efficiently found by the min-cut max-flow algorithm. However, image segmentation using minimum cut has a bias toward producing shorter boundaries [15].

Graph Cuts with Normalized Costs

Various kinds of normalization have been incorporated in graph-cut cost functions to reduce the bias toward producing shorter boundaries. Shi and Malik [12] developed a *normalized-cut* approach with a cost function

$$\frac{c(G_1, G_2)}{c(G_1, G)} + \frac{c(G_1, G_2)}{c(G_2, G)},$$

which favors two subgraphs with similar total edge weights and, therefore, avoids producing overly short boundaries. Finding the globally optimal normalized cut is a computationally intractable NP-hard problem. In practice, an approximate solution can be obtained using spectral graph theory, by relaxing this discrete optimization problem to the continuous domain.

Wang and Siskind [14] developed a *ratio-cut* approach with a cost function

$$\frac{c(G_1, G_2)}{\#(G_1, G_2)},$$

where $\#(G_1, G_2)$ is the number of edges connecting G_1 and G_2. This reflects the average weight along a cut boundary and, therefore, reduces the bias toward short boundaries. The globally optimal ratio cut can be found efficiently in polynomial time. Figure 2 shows sample segmentation results using normalized cut and ratio cut.

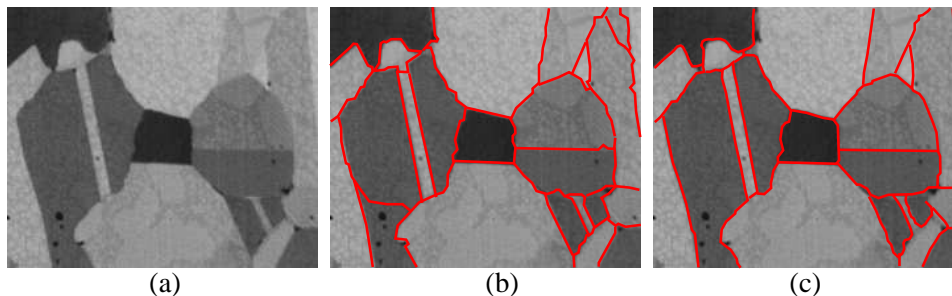(a)   (b)   (c)

Figure 2: Segmentations on a (cropped) Ni-based alloy image (a) using normalized cut (b) and ratio cut (c).

Similar to normalized cut is the *average-cut* approach [11], where the size of subgraphs G_1 and G_2 is used for normalization. Finding the optimal average cut is NP-hard and an approximate solution can be obtained using spectral graph theory. Cox, Rao, and Zhong [3] developed a *ratio-region* approach for image segmentation, which handles only the closed-cut boundary as shown in Fig. 1(d) and uses the area within the closed-cut boundary for normalization. Jermyn and Ishikawa [6] extended the ratio-region approach by considering region information other than the area. The optimal ratio region and its extension in [6] can be found efficiently in polynomial time. Many other graph-cut methods have also been developed for image segmentation [13, 5, 9, 4].

Segmentation with a Template

Many materials science images are in the form of a sequence of 2D serial sections (i.e., slices). The grain structure between neighboring slices usually shows good consistency and continuity. Therefore, we can use

the segmentation of one slice as a template to guide the segmentation of its neighboring slices. Graph-cut methods, particularly minimum cut, can be used for this purpose.

Specifically, image segmentation can be formulated as an assignment of labels to each pixel where the pixels with the same label constitute a segment. In [1, 7], the objective of optimal labeling is to find a labeling function f by minimizing the cost function

$$E(f) = \sum_{p \in \mathcal{P}} D_p(f_p) + \sum_{\{p,q\} \in \mathcal{N}} V_{pq}(f_p, f_q),$$

where $\mathcal{P}$ is the set of image pixels, f_p is the label of pixel $p \in \mathcal{P}$, and $\mathcal{N}$ is the set of pixel pairs that are neighbors in the image. The data term $D_p(f_p)$ describes the cost of assigning label f_p to pixel p and the smoothness term $V_{pq}(f_p, f_q)$ describes the cost of assigning labels f_p and f_q to two neighboring pixels p and q, respectively. Finding the globally optimal labeling that minimizes this cost function is NP-hard. However, the minimum graph-cut algorithm can be used to obtain a locally optimal labeling efficiently.

In this optimal labeling framework, we can define a specialized data term and smoothness term to enforce the grain structure consistency when propagating an image segmentation from one slice to another. For example, in defining the data term, we can set an infinity cost $D_p(f_p)$ when label f_p is not assigned to any pixel near p in the template. In defining the smoothness term, we can set an infinity cost $V_{pq}(f_p, f_q)$ if the segments with labels f_p and f_q are not adjacent in the template. Figure 3(c) shows a sample segmentation result using this method. Various kinds of human interaction, such as the selection of seed points for individual grains, can be effectively and conveniently integrated into this framework [2].

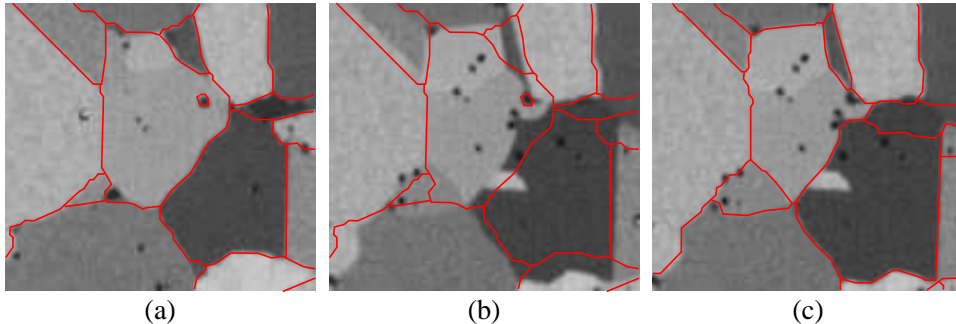| (a) | (b) | (c) |

Figure 3: Segmentation of a (cropped) Ni-based alloy image by optimal labeling. (a) Segmented grain boundaries in a template slice. (b) Grain boundaries in (a) copied to a new slice. (c) Segmented grain boundaries in this new slice after optimal labeling.

Conclusion

Graph-cut methods have been successfully used in many image-segmentation applications. Compared with other image segmentation methods, graph-cut methods employ global cost functions, attempt to find globally optimal solutions to these cost functions, and have the capability of considering both image information, such as intensity, and structure information, such as a template. These properties make graph-cut methods a very promising solution to the challenging problem of grain boundary segmentation.

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